Duality Transformations Away From Conformal Points*

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Abstract

Target space duality transformations are considered for bosonic sigma models and strings away from RG fixed points. A set of consistency conditions are derived, and are seen to be nontrivially satisfied at one-loop order for arbitrary running metric, antisymmetric tensor and dilaton backgrounds. Such conditions are sufficiently stringent to enable an independent determination of the sigma model beta functions at this order.

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1. Introduction

Target space duality symmetry ('T-duality', or simply 'duality' henceforth) was first observed for strings compactified on a torus, where the partition function could be computed and explicitly verified to be invariant under the transformation $R \to 1/R$, where R is the target radius [1]. On these and some other particular backgrounds, e.g. WZW models, duality has in fact been studied in great detail as a symmetry of the full quantum theory [2]. However, even for arbitrary backgrounds with an abelian isometry, and when the partition function is not known explicitly, there is a proof of invariance under T-duality transformations which comes directly from the sigma model path integral [3]. In this proof, all manipulations are done on classical background fields, and essentially no effect of renormalization is taken into account. Duality appears as a classical symmetry, and there is not really a path integral reasoning connecting it to the quantum properties of the sigma model. Naturally, if one starts out with a conformally invariant background, then it is reasonable to expect that duality will also be a symmetry at the quantum level, since there will not be any perturbative quantum corrections to the background. At one-loop order this expectation is in fact borne out, and conformally invariant backgrounds are mapped by classical duality into conformally invariant backgrounds [3]. Nevertheless, conformal backgrounds are a very small subset of all possible backgrounds, and one might wonder whether it is possible to extend the action of duality beyond these very special cases. While for string theory the main interest may lie in conformal backgrounds, from a pure 2d field theory point of view, we find it would be interesting to examine the interplay between duality and RG flows. This represents the main motivation underlying our investigations here.

Our starting point is a simple and basic observation: any manipulations on a path integral can only be meaningful once the path integral itself is well-defined, and for that one must consider it with proper regularization and renormalization in place. The standard manipulations on classical fields leading to duality transformations, which may be justifiable for conformal backgrounds, should more appropriately be considered on the renormalized background fields in general. Once this is done, duality transformations will still have the same form, but now they act on renormalized fields which, in particular, flow with a renormalization group parameter μ. As such, duality then maps entire renormalization flows into other renormalization flows and thus, like other symmetries in field theory, it also points to the fact that our parametrization of a certain field space is redundant.

More importantly, once one considers this flow in the duality transformations, we will show that a stringent set of consistency conditions follows on the possible quantum corrections in order that they respect duality symmetry. These conditions are expressed as linear homogeneous relations among the beta functions of the original and dual theories.

We will investigate the validity of these consistency relations in all generality at one-loop order. The result we find is that they are satisfied for entirely generic backgrounds of metric, antisymmetric tensor and dilaton, due to the specific form the beta functions take. In fact, as we shall see explicitly, one can turn the argument around and actually *derive* the one-loop beta functions simply by requiring the consistency relations.

An important distinction should be made between the sigma model on a flat worldsheet (or simply 'sigma model' in what follows) and the string, in order to make more precise what we have been loosely referring to as 'beta functions' in the above. While for the sigma model the quantities appearing in the consistency relations are the beta functions, in the string, there is an extra background field, the dilaton, and the relevant quantities are the Weyl anomaly coefficients [4]. In this latter case, the consistency relations will only hold provided the well-known dilaton shift [3] is also implemented. Apart from this, the difference that occurs between the two cases is that the relations will be exactly satisfied for the Weyl anomaly coefficients, while the sigma model beta functions will only satisfy them after a particular field redefinition. At conformal points of the string, in particular, this will imply the well-known statement that conformal backgrounds are mapped to conformal backgrounds under duality. The analogous statement for the sigma model is that scale invariant (or on-shell finite) backgrounds will be mapped to scale invariant backgrounds.

In what follows, we will initially write down the sigma model under consideration, namely, on a generic metric and antisymmetric tensor background, and the duality transformations that ensue from the abelian isometry being assumed for the model. We will then derive the consistency relations that follow for the quantum corrections to the background.

Both the duality transformations and the consistency relations treat the background in a way which breaks manifest target space covariance. In order to deal with this, we will verify the consistency relations through a Kaluza-Klein decomposition of the background tensors. We will present the well-known results for the one-loop beta functions and Weyl anomaly coefficients [4], and then proceed to perform the decomposition on these quantities. When this is done, it is then a lengthy but straightforward exercise to show that the relations are indeed satisfied without restrictions on the background. We will explicitly see how the dilaton shift re-emerges in this context, as well as the specific $O(\alpha')$

field redefinition necessary for the sigma model beta functions. It will also become clear that the mixing of torsion and geometry entailed by duality is exactly "matched" by the precise admixture of torsion and geometry in the one-loop beta functions. This is what allows for an independent calculation of the beta functions at this order starting from our consistency requirements.

2. Duality Transformations and Consistency Relations

Our starting point is the $d = 2$ bosonic sigma model in a generic $D+1$-dimensional background $\{g_{\mu\nu}(X), b_{\mu\nu}(X)\}$ of metric and antisymmetric tensor, respectively, where $\mu, \nu = 0, 1, \ldots, D = 0, i$, so that the $\mu = 0$ component is singled out. We shall assume this sigma model has an abelian isometry, which will enable duality transformations, and we shall consider the background above in the adapted coordinates, in which the abelian isometry is made manifest through independence of the background on the coordinate $\theta \equiv X^0$ [3]. The original sigma model action reads:

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[g_{00}(X)\partial_\alpha\theta\partial^\alpha\theta + 2g_{0i}(X)\partial_\alpha\theta\partial^\alpha X^i + g_{ij}(X)\partial_\alpha X^i\partial^\alpha X^j + \right.$$
$$\left. i\varepsilon^{\alpha\beta}\left(2b_{0i}(X)\partial_\alpha\theta\partial_\beta X^i + b_{ij}(X)\partial_\alpha X^i\partial_\beta X^j\right)\right] . \tag{2.1}$$

Not only in the above, but throughout, all background tensors can depend only on target coordinates X^i, $i = 1, \ldots, D$, and not on θ.

Regularization and renormalization of the sigma model is achieved typically through dimensional regularization plus background field method plus normal coordinate expansion [7]. Upon such regularization a quantum effective action is obtained, in which the background metric and antisymmetric tensor become functions of a renormalized scale μ through quantum corrections in the form of curvature and torsion terms. Such a procedure is standard and well-known, and we will assume it here *ab initio*.

The duality transformations in this model are also well-known [3]:

$$\tilde{g}_{00} = \frac{1}{g_{00}}$$
$$\tilde{g}_{0i} = \frac{b_{0i}}{g_{00}} , \quad \tilde{b}_{0i} = \frac{g_{0i}}{g_{00}}$$
$$\tilde{g}_{ij} = g_{ij} - \frac{g_{0i}g_{0j} - b_{0i}b_{0j}}{g_{00}} \tag{2.2}$$
$$\tilde{b}_{ij} = b_{ij} - \frac{g_{0i}b_{0j} - b_{0i}g_{0j}}{g_{00}} .$$

The statement of classical duality is that the model defined on the dual background $\{\tilde{g}_{\mu\nu}, \tilde{b}_{\mu\nu}\}$ is simply a different parametrization of the same model, given that the manipulations used to derive the transformations essentially only involve performing trivial integrations in a different order starting from the path-integral in which the abelian isometry is gauged.

Our further observation here, which represents the starting point of this investigation, is that such manipulations must be considered on a properly regularized path integral, with bare background fields containing all the necessary counterterms *in lieu* of the classical fields (one should also assure oneself that once the classical fields have a certain isometry manifest in a certain coordinate system, so will the bare fields including all perturbative corrections. A moment's thought shows this is generically true). Once this is done, the usual procedure of interchanging the order of integrations in the gauged model [3] will lead to two quantum effective actions, one based on the original background, and one based on the dual one. The duality transformations remain the same as the ones above, but all quantities should be taken to be a function of a renormalization scale μ, so that their flow under changes in μ succintly take into account the quantum corrections they contain. Denoting by

$$\beta^g_{\mu\nu} \equiv \mu \frac{d}{d\mu} g_{\mu\nu} \,, \quad \beta^b_{\mu\nu} \equiv \mu \frac{d}{d\mu} b_{\mu\nu} \tag{2.3}$$

the metric and antisymmetric tensor beta functions, respectively, and applying $\mu d/d\mu$ to the duality transformations, one obtains:

$$\beta^{\tilde{g}}_{00} = -\frac{1}{g_{00}^2} \beta^g_{00}$$

$$\beta^{\tilde{g}}_{0i} = -\frac{1}{g_{00}^2} \left(b_{0i} \beta^g_{00} - \beta^b_{0i} g_{00} \right)$$

$$\beta^{\tilde{b}}_{0i} = -\frac{1}{g_{00}^2} \left(g_{0i} \beta^g_{00} - \beta^g_{0i} g_{00} \right) \tag{2.4}$$

$$\beta^{\tilde{g}}_{ij} = \beta^g_{ij} - \frac{1}{g_{00}} \left(\beta^g_{0i} g_{0j} + \beta^g_{0j} g_{0i} - \beta^b_{0i} b_{0j} - \beta^b_{0j} b_{0i} \right) + \frac{1}{g_{00}^2} \left(g_{0i} g_{0j} - b_{0i} b_{0j} \right) \beta^g_{00}$$

$$\beta^{\tilde{b}}_{ij} = \beta^b_{ij} - \frac{1}{g_{00}} \left(\beta^g_{0i} b_{0j} + \beta^b_{0j} g_{0i} - \beta^g_{0j} b_{0i} - \beta^b_{0i} g_{0j} \right) + \frac{1}{g_{00}^2} \left(g_{0i} b_{0j} - b_{0i} g_{0j} \right) \beta^g_{00} \,,$$

where the quantities on the l.h.s. are the beta functions of the dual background. These are then the consistency relations that the beta functions must satisfy in order that quantum corrections in the sigma model satisfy classical duality symmetry. The above conditions may also be seen as a statement of "covariance" of the renormalization group flow under

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duality transformations. It should be noted that the above relations imply highly nontrivial and restrictive constraints, since classical duality transformations have in principle no information on what the actual renormalization of the theory is.

On a generic background, the one-loop beta functions are [5],[6],[4]:

$$
\begin{aligned}
\beta^g_{\mu\nu} &= \alpha' \left(R_{\mu\nu} - \frac{1}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} \right) \\
\beta^b_{\mu\nu} &= -\frac{\alpha'}{2} \nabla_\lambda H^\lambda{}_{\mu\nu} \;,
\end{aligned}
\tag{2.5}
$$

where $H_{\mu\nu\lambda} = \partial_\mu b_{\nu\lambda} +$ cyclic permutations, and ∇_μ denotes the torsionless covariant derivative. We must verify that these beta functions satisfy the conditions above if the original and dual backgrounds are related as in (2.2). We will also consider the analogous relations for the Weyl anomaly coefficients, which are given at one loop by [4]:

$$
\begin{aligned}
\bar\beta^g_{\mu\nu} &= \beta^g_{\mu\nu} + 2\alpha' \nabla_\mu \partial_\nu \phi \\
\bar\beta^b_{\mu\nu} &= \beta^b_{\mu\nu} + \alpha' H_{\mu\nu}{}^\lambda \partial_\lambda \phi \;.
\end{aligned}
\tag{2.6}
$$

3. Kaluza-Klein Decomposition

In order to verify identities which break manifest target space covariance in one direction, we find that the most direct and economical way to proceed is to consider the decomposition of tensors which is typical of Kaluza-Klein reductions. We write the arbitrary metric $g_{\mu\nu}$ as:

$$
g_{\mu\nu} = \begin{pmatrix} a & av_i \\ av_i & \bar g_{ij} + av_i v_j \end{pmatrix} \;,
\tag{3.1}
$$

so that $g_{00} = a, g_{0i} = av_i, g_{ij} = \bar g_{ij} + av_i v_j$, and all quantities do not depend on $X^0 = \theta$. The components of the antisymmetric tensor are also decomposed as $b_{0i} \equiv w_i$ and b_{ij}. Under (2.2), the dual background is easily found to be:

$$
\tilde g_{\mu\nu} = \begin{pmatrix} 1/a & w_i/a \\ w_i/a & \bar g_{ij} + w_i w_j/a \end{pmatrix} \;,
\tag{3.2}
$$

and $\tilde b_{0i} = v_i, \tilde b_{ij} = b_{ij} + w_i v_j - w_j v_i$.

We now need to work out the expression for the connection coefficients and Ricci tensor for both original and dual geometries, but of course we only need to do it once, since the dual geometry is obtained from the original one by the substitution $a \to 1/a, v_i \to w_i$.

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Likewise, dual torsion is obtained from the original one by $w_i \to v_i$ and $b_{ij} \to b_{ij} + w_i v_j - w_j v_i$. We list below all geometric quantities relevant for our computation:

1) *inverse metric*: $g^{00} = 1/a + v_i v^i$, $g^{0i} = -v^i$, $g^{ij} = \bar{g}^{ij}$. On decomposed tensors, indices $i, j, \ldots$ are raised and lowered with the metric $\bar{g}_{ij}$ and its inverse. We note also that $\det g = a \det \bar{g}$.

2) *connection coefficients*:

$$
\begin{aligned}
\Gamma^0_{00} &= \frac{a}{2} v^i a_i \ , \quad \Gamma^0_{i0} = \frac{a}{2}\left[\frac{a_i}{a} + v^j a_j v_i + v^j F_{ji}\right] \\
\Gamma^i_{00} &= -\frac{a}{2} a^i \ , \quad \Gamma^i_{0j} = -\frac{a}{2}\left[F^i{}_j + a^i v_j\right] \\
\Gamma^0_{ij} &= -\bar{\Gamma}^k_{ij} v_k + \frac{1}{2}(\partial_i v_j + \partial_j v_i + a_i v_j + a_j v_i) - \frac{a}{2} v^k \left[v_j F_{ik} + v_i F_{jk} - a_k v_i v_j\right] \\
\Gamma^i_{jk} &= \bar{\Gamma}^i_{jk} + \frac{a}{2}\left[v_j F_k{}^i + v_k F_j{}^i - a^i v_j v_k\right] \ ,
\end{aligned}
\tag{3.3}
$$

where $a_i = \partial_i \ln a$, $F_{ij} = \partial_i v_j - \partial_j v_i$, and $\bar{\Gamma}^i_{jk}$ are the connection coefficients for the metric $\bar{g}_{ij}$.

3) *Ricci tensor*:

$$
\begin{aligned}
R_{00} &= -\frac{a}{2}\left[\bar{\nabla}_i a^i + \frac{1}{2} a_i a^i - \frac{a}{2} F_{ij} F^{ij}\right] \\
R_{0i} &= v_i R_{00} + \frac{3a}{4} a^j F_{ij} + \frac{a}{2}\bar{\nabla}^j F_{ij} \\
R_{ij} &= \bar{R}_{ij} + v_i R_{0j} + v_j R_{0i} - v_i v_j R_{00} - \frac{1}{2}\bar{\nabla}_i a_j - \frac{1}{4} a_i a_j - \frac{a}{2} F_{ik} F_j{}^k \ ,
\end{aligned}
\tag{3.4}
$$

where, again, barred quantities refer to the metric $\bar{g}_{ij}$.

4) *torsion*:

$$
\begin{aligned}
H_{0ij} &= -\partial_i w_j + \partial_j w_i \equiv -G_{ij} \\
H_{ijk} &= \partial_i b_{jk} + \partial_j b_{ki} + \partial_k b_{ij} \ ,
\end{aligned}
\tag{3.5}
$$

and all other components vanish. For the metric beta function we need

$$
\begin{aligned}
H_{0\mu\nu} H_0{}^{\mu\nu} &= G_{ij} G^{ij} \\
H_{0\mu\nu} H_i{}^{\mu\nu} &= -2 G_{ij} G^{jk} v_k - H_{ijk} G^{jk} \\
H_{i\mu\nu} H_j{}^{\mu\nu} &= 2\left(\frac{1}{a} + v_m v^m\right) G_i{}^k G_{jk} - 2 v^k v^m G_{ik} G_{jm} + 2\left(H_{ikm} G_j{}^k v^m + i \leftrightarrow j\right) \\
&\quad + H_{ikm} H_j{}^{km} \ ,
\end{aligned}
\tag{3.6}
$$

and for the antisymmetric tensor beta function we need

$$\nabla_\mu H^\mu{}_{0i} = \bar{\nabla}^j G_{ji} - a G_{ij} F^{jk} v_k + \frac{1}{2} G_{ij} a^j - \frac{a}{2} F^{jk} \left(H_{ijk} + v_i G_{jk} \right)$$

$$\nabla_\mu H^\mu{}_{ij} = \bar{\nabla}^k \left(H_{kij} + v_k G_{ij} \right) - \frac{1}{2} \left[G_i{}^k \bar{\nabla}_{(k} v_{j)} - G_j{}^k \bar{\nabla}_{(k} v_{i)} \right]$$
$$- \frac{a}{2} v_{[i} H_{j]km} F^{km} + v_{[i} G_{j]k} \left(a^k - a F^{km} v_m \right)$$
$$+ \frac{1}{2} a^k H_{kij} + \frac{1}{2} v_m a^m G_{ij} - \frac{1}{2} F_{[i}{}^k G_{j]k} \; ,$$

$$(3.7)$$

where $[ij] = ij - ji$ and $(ij) = ij + ji$.

5) *dilaton terms*:

$$\nabla_0 \partial_0 \phi = \frac{a}{2} a^i \partial_i \phi$$

$$\nabla_0 \partial_i \phi = \frac{a}{2} \left(F^j{}_i + a^j v_i \right) \partial_j \phi$$

$$(3.8)$$

$$\nabla_i \partial_j \phi = \bar{\nabla}_i \partial_j \phi - \frac{a}{2} \left(v_i F_j{}^k + v_j F_i{}^k - a^k v_i v_j \right) \partial_k \phi \; .$$

For the dual background, we take $\phi \to \tilde{\phi}$, with $\tilde{\phi}$ as yet undetermined. Consistency conditions will tell us what it is.

One must also compute the analogous quantities in the dual background, through the substitutions $a \to 1/a$, $v_i \leftrightarrow w_i$, $b_{ij} \to b_{ij} + w_i v_j - w_j v_i$. This is straightforward, and we will not do it here. These results (and quite some patience!) are essentially all one needs to verify consistency relations (2.4) at one-loop order both for Weyl anomaly coefficients and beta functions. We show this explicitly for the first and simplest one (the 00 component), since it already presents all the essential features we have alluded to above. Starting with the Weyl anomaly coefficient:

$$\bar{\beta}_{00}^{\tilde{g}} = \tilde{R}_{00} - \frac{1}{4} \tilde{H}_{0\lambda\rho} \tilde{H}_0{}^{\lambda\rho} + 2 \tilde{\nabla}_0 \partial_0 \tilde{\phi}$$

$$= - \frac{1}{2a} \left(-\bar{\nabla}_i a^i + \frac{1}{2} a_i a^i - \frac{1}{2a} G^2 \right) - \frac{1}{4} F^2 - \frac{1}{a} a^i \partial_i \tilde{\phi}$$

$$-\frac{1}{g_{00}^2} \bar{\beta}_{00}^g = - \frac{1}{g_{00}^2} \left(R_{00} - \frac{1}{4} H_{0\lambda\rho} H_0{}^{\lambda\rho} + 2 \nabla_0 \partial_0 \phi \right)$$

$$= - \frac{1}{a^2} \left[-\frac{a}{2} \left(\bar{\nabla}_i a^i + \frac{1}{2} a_i a^i - \frac{a}{2} F^2 \right) - \frac{1}{4} G^2 + a a^i \partial_i \phi \right] \; ,$$

$$(3.9)$$

with $G^2 \equiv G_{ij} G^{ij}$ and $F^2 \equiv F_{ij} F^{ij}$ (we have set $\alpha' = 1$ since it is unimportant here). These two expressions will only match once we make the identification $\tilde{\phi} = \phi - \frac{1}{2} \ln a$. This reproduces the well-known dilaton shift present in duality transformations.

The analogous relation for the beta function is:

$$
\begin{aligned}
\beta_{00}^{\tilde{g}} &= \tilde{R}_{00} - \frac{1}{4}\tilde{H}_{0\lambda\rho}\tilde{H}_0{}^{\lambda\rho} \\
&= -\frac{1}{2a}\left(-\bar{\nabla}_i a^i + \frac{1}{2}a_i a^i - \frac{1}{2a}G^2\right) - \frac{1}{4}F^2 \\
-\frac{1}{g_{00}^2}\beta_{00}^{g} &= -\frac{1}{g_{00}^2}\left(R_{00} - \frac{1}{4}H_{0\lambda\rho}H_0{}^{\lambda\rho}\right) \\
&= -\frac{1}{a^2}\left[-\frac{a}{2}\left(\bar{\nabla}_i a^i + \frac{1}{2}a_i a^i - \frac{a}{2}F^2\right) - \frac{1}{4}G^2\right] \;,
\end{aligned}
\tag{3.10}
$$

(with $\alpha' = 1$ here again). We immediately realize that since the dilaton is not present, neither is the dilaton shift which was necessary previously for consistency, and the two expressions above do not match. However, an $O(\alpha')$ field redefinition coming from a target reparametrization [8] in the original model cures this mismatch:

$$
\beta_{\mu\nu}^{g} \to \beta_{\mu\nu}^{'g} = \beta_{\mu\nu}^{g} - \alpha'\nabla_{(\mu}\,\xi_{\nu)} \;,
\tag{3.11}
$$

with $\xi_\mu = -1/2\,\partial_\mu \ln a$, so that

$$
\beta_{00}^{'g} = \beta_{00}^{g} + \frac{\alpha'}{2}aa_i a^i \Rightarrow \beta_{00}^{\tilde{g}} = -\frac{1}{g_{00}^2}\beta_{00}^{'g} \;,
\tag{3.12}
$$

so that consistency is again verified. It is worthwhile noting that this fits nicely with the fact that while the string has a dilaton and the sigma model does not, the Weyl anomaly coefficients do not transform under field redefinitions, but the beta functions do [4]. An analogous mismatch will occur in the relations in which the antisymmetric tensor beta function is involved. It will again be removed with the same field redefinition as above, where now the antisymmetric beta function also changes to

$$
\beta_{\mu\nu}^{b} \to \beta_{\mu\nu}^{'b} = \beta_{\mu\nu}^{g} - \alpha' H_{\mu\nu}{}^{\lambda}\xi_\lambda
\tag{3.13}
$$

up to a gauge transformation. Recalling that scale invariance of the sigma model (which is equivalent to on-shell finiteness) requires that the beta functions be

$$
\begin{aligned}
\beta_{\mu\nu}^{g} &= \nabla_{(\mu}\,V_{\nu)} \\
\beta_{\mu\nu}^{b} &= H_{\mu\nu}{}^{\lambda}V_\lambda \;,
\end{aligned}
\tag{3.14}
$$

for V_μ some target vector, the above then shows that under classical duality a one-loop finite sigma model will be mapped to a one-loop finite sigma model.

We finally note that while the G^2 term in (3.10) comes from geometry in the dual model, it comes from torsion in the original one, and vice-versa for the F^2 term. On the other hand, we know from scaling arguments [9][7] that a one-loop metric beta function must be a linear combination of the geometric quantities $R_{\mu\nu}$, $H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho}$, $g_{\mu\nu}R$ and $g_{\mu\nu}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma}$. Because of this "interchange" of torsion and geometry, if we take an arbitrary linear combination of these four terms for the metric beta function and require the above consistency conditions, we find a relative factor $-1/4$ between the first two terms, while the last two terms, $g_{\mu\nu}R$ and $g_{\mu\nu}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma}$, have vanishing coefficients (this is found already for the 00 component).[†] This same interchange of torsion and geometry between the original and dual backgrounds will take place in all of equations (2.4) (but in a considerably more involved way than simply $G^2 \leftrightarrow F^2$ for the other relations). The other relations contain furthermore terms coming from $R_{\mu\nu}$ matching terms coming from $\nabla_\lambda H^\lambda{}_{\mu\nu}$. This matching fixes their relative factor of $-1/2$. Antisymmetry in $\mu\nu$ and the same scaling arguments lead to this latter quantity as the only possible one-loop antisymmetric beta function, and altogether then, the one-loop metric and antisymmetric tensor beta functions are completely determined up to a global constant (which of course cannot be determined from these relations since they are linear and homogeneous).

Components $0i$ of (2.4) require the matching of roughly 20 different terms, while components ij involve approximately 80 different terms. We will not present any details of these calculations here, but rather just mention that they exactly corroborate the claims we have made through examination of the 00 component alone.

4. Conclusions

In this Letter we have considered the action of T-duality transformations on backgrounds away from conformal points in order to study the interplay between classical duality symmetry and RG flows. A set of consistency relations follow for the quantum corrections to the sigma model which are satisfied in all generality at one-loop order, showing that RG flows are entirely compatible with duality at this order. At limiting points of the flows, the consistency relations state that for the sigma model, scale invariant backgrounds are mapped to scale invariant backgrounds, while for the string, the well-known statement

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[†] We thank D.Z. Freedman and R.C. Myers for reminding us that scaling arguments do not rule out the terms $g_{\mu\nu}R$ and $g_{\mu\nu}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma}$ from the one-loop metric beta function.

that conformally invariant backgrounds are mapped to conformally invariant backgrounds is recovered. These consistency relations are stringent enough to allow for an independent determination of the one-loop beta functions up to one global factor.

There have been to date a few investigations on the issue of preservation of classical duality symmetry on conformal backgrounds at two-loop order, and possible corrections to the classical transformations [10][11][12]. Ref. [10], in particular, dealing with the more restricted case of torsionless original and dual backgrounds, proposes a correction to the transformations in order to preserve duality symmetry of the two-loop string low energy effective action. Such corrections probably indicate subtleties regarding duality transformations which are not entirely accounted for in the standard path integral derivation in [3]. Any such perturbative modifications of the transformations at the conformal point are likely to engender modifications in the transformations on the running couplings as well, and thus on the consistency relations which follow from these. It would be interesting to examine consistency relations in the presence of such modifications. Such investigations, based mainly on cases treated in [10] and [12], are currently in progress.

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